Quantum mechanics and observation on macroscopic arrows

A. Dijksman

Abstract

Ordinary arrows or needles are represented mathematically by vectors. Observations on such objects may therefore be treated using a vector space framework. An example is given where rotating needles impinge on a wire grid and where the measurement result is analyzed using state vectors in a two-dimensional complex vector state space where the usual quantum rules apply. It is argued that Bell's inequality tests concern only classical analogies where the measurement result characterizes univocally the state of the system. Deeper investigation on the interaction and spinning modes of arrows suggest other analogies with the properties of the fundamental particles.

Keywords

Quantum measurement, photon polarization, quantum analogies.

PACS

03.65.Ta - Foundations of quantum mechanics; measurement theory.

1. Introduction

What exactly does a measurement result learn us about an object? Addressing this question, Bell suggests that "when it is said that something is 'measured' it is difficult not to think of the result as referring to some pre-existing property of the object in question"[1]. However, as he further points out, obtaining a measurement result involves the object (system) *and* the apparatus, in agreement with Bohr's interpretation. It would therefore be more appropriate to say that the result refers to a contextual property of the object-apparatus *interaction*, as opposed to some pre-existing property attached solely to the object. For example, in the case of a position measurement, the result refers to the location of the point of interaction, which realistically rarely coincides with the *mathematical* location of the object, unless the object is classically reduced to a point. There is an intrinsic indeterminacy in the result of elementary position observations, due to the geometrical extent of objects. In a classical scheme, one completes the measurement by some calculations including results from other observations (mass, contour, density of the object), in order to yield a presentable result, say the location of the center of gravity, which then stands for the exact location of the object.

In the conceptual development of Quantum Mechanics, the classical assumption that the exact value of physical properties may in principle be obtained by measurement is generally taken for granted. The following quotes of founding texts illustrate this: "There is no shortage of such experiments, which in principle even allow one to determine the *position of the electron* with arbitrary accuracy" [2]. "If the dynamical system is in an eigenstate of a real dynamical variable ξ belonging to the eigenvalue ξ', then a measurement of ξ will certainly give as result the number ξ' " [3]. And EPR referring to the wavefunction equation of a particle in a state of definite momentum: "A definite value of the coordinate, for a particle in the state given by Eq. (2), is thus not predictable, but may be obtained only by a direct measurement." [4]. However reasonable this assumption may be in a classical scheme, a realistic scheme allows a broader view on the question. If an elementary particle has geometrical extent, which seems to be a realistic hypothesis as regard to common experience, measured location refers to the interaction location, which has an indetermination proportional to the extent of the elementary particle.

This viewpoint developed in the following sections leads to an alternative view on the domain of applicability of quantum mechanics. Section 2 presents an experimental situation with ordinary objects that may be treated quantum mechanically. Section 3 questions the relevance of such an analogy with respect to Bell's inequality test. Section 4 briefly suggests an extension of the investigational field on the hand of the general motion of ordinary needles or arrows, before concluding in section 5 that the interpretation of quantum physics may gain clarity if quantum analogies with ordinary objects are further developed.

2. Naive quantum analogy

A quantum vector space scheme emerges quite naturally if one assumes that elementary particles have one-dimensional geometrical extent. Such particles may be represented mathematically as oriented rectilinear segments, i.e. vectors. Spaces in which vectors evolve are by definition vector spaces. Which set of vectors is chosen as spanning basis for its structuration is a matter of convenience and depends on the context. As an illustration, let us describe quantum mechanically the physics of one-dimensional identical rectilinear (macroscopic) objects, say thin needles, on which we perform a specific two-valued observation. We shoot them with velocity $\boldsymbol{v}$ in the horizontal z-direction towards a wire grid, whose infinite parallel thin wires, oriented along the vertical x-direction, are regularly interspaced by a distance equal to the length of the needles, which we take as our unity, cf. Fig. 1. We let the shooting device at z=0 impart the needles with a plane rotational spinning motion whose normal angular velocity vector $\boldsymbol{\omega}$ is orthogonal to $\boldsymbol{v}$, with $\boldsymbol{v} >> \boldsymbol{\omega}$. Let t_g denote the instant that the needle reaches the grids plane. We finally define the specific observable $\boldsymbol{P}$ to take the value 1 if the needle crosses the grid without affecting it, 0 if the needle collides with a wire.

Quantum mechanically, it is convenient to make each basis vector correspond to a definite value of the observable. If the needle is spinning in the *x-z* plane and because the needle and the wires are supposed to be idealistically thin, the needle will never collide with the wires and the observable $\boldsymbol{P}$ will always yield the eventuality 1. A needle spinning in the *y-z* plane and that reaches the grid plane, while oriented along the *y*-direction, yields always the eventuality 0. If the needle reaches the wire grid in another configuration, the value of the observable is either 0 or 1. With θ the angle between the needle's rotational plane and the *x-z* plane and φ the angle between the *z*-axis and the needle's direction, the probability that the needle collides with a wire ($\boldsymbol{P}$=0) is equal to the projection of the needle on the *y*-axis, when it intersects the grid. The length of this projection is given by $\sin\theta\,\sin\varphi$. The probability for $\boldsymbol{P}$=1 is then given by 1 - $\sin\theta\,\sin\varphi$. Let now the orientation of the needle be steered by a pilot wave in such a way that the condition $\theta = \varphi$ always holds at the grid's plane. Because of this pilot wave constraint, this specific experiment becomes an analogy of a polarization measurement of photons, with Prob($\boldsymbol{P}$=1) = $\cos^2\theta$ and Prob($\boldsymbol{P}$=0) = $\sin^2\theta$ in conformity with Malus' law.

The two states with definite value then suggest a two-dimensional vector state space, with spanning vectors:

- the state vector |1> = (1 0) representing the needle spinning in the *x-z* plane (which we denote as an *x*-polarized needle), corresponding to a definite value $\boldsymbol{P}$=1,
- the state vector |0> = (0 1) representing the *y*-polarized needle, corresponding to a definite value $\boldsymbol{P}$=0.

The usual quantum rules then apply to the vector space with basis vectors |1> and |0>.

If |1> corresponds to the *x*-polarized needle arriving at the grid's plane with φ=0, the same vector rotated by $\gamma=\omega(t_g-t)$ in the polarization plane corresponds to the needle at an instant $t<t_g$. This phase change is represented by the multiplication of the state vector by the complex scalar $\exp(i\gamma)$. With respect to the considered two-valued observation the state vector $\exp(i\gamma)$|1> then of course represents the same state as |1>.

The linear combination of two basis states results in another state provided that it remains normalized to unity. For example, the needle rotating in the polarization plane of angle θ with the *x-z* plane is described by the state vector $|\theta>$:

$$|\theta\rangle = \left(i\cos\theta|1\rangle + \sin\theta|0\rangle\right)/\sqrt{2} \qquad (1)$$

The factor exp($i\pi/2$) is necessary to adjust the phase of the basis vector |1> in order to yield a linear polarization for $|\theta>$. The combination means that a needle in the intermediate state $|\theta>$ has some chance that the polarization measurement results in the eventuality 1 and some chance that it results in the eventuality 0. Probabilities for the chances are obtained by taking the absolute value of the squared complex scalar coefficient of the corresponding basis vector (the amplitude). The probability that $|\theta>$ is observed in state |1> could also be written $<\theta|1><1|\theta>$ where $<\psi|\varphi>$ denotes the scalar product defined over the vector space. $<\psi|\varphi>$ yields always a complex scalar $a\exp(i\gamma)$, with a and γ real, where γ measures the phase difference for $|\varphi>$ projected onto $|\psi>$ and where a is the product of the length of $|\psi>$ and the length of the projection of $|\varphi>$ on $|\psi>$. $<\varphi|\psi>$ then stands for the complex conjugate $a\exp(-i\gamma)$.

The phase change of the state vector corresponding to the needle rotating with angular velocity ω is expressed by the factor $\exp(i\omega\Delta t)$ multiplying the state vector, giving therefore the following equation:

$$|\Psi(t+\Delta t)> = \exp(i\omega\Delta t)\,|\Psi(t)>. \qquad (2)$$

At the limit $\Delta t \rightarrow 0$, the vector difference between $|\Psi(t+\Delta t)>$ and $|\Psi(t)>$, when represented in real space, has length $\omega\Delta t$ ($|\Psi>$ is normalized to unity) and is perpendicular to $|\Psi(t)>$, corresponding to a phase change of $\pi/2$, i.e. multiplication by the factor i, giving the differential equation:

$$d|\Psi(t)>/dt = i\omega|\Psi(t)>. \qquad (3)$$

It is possible to reproduce the behavior of a photon crossing successive polaroid sheets whose optical axes are at an angle different from zero: when the wire grid is completed by a mechanism that stops the colliding needles and rotates the initial polarization plane into the analyzing plane (the *x-z* plane of the example) for those needles that cross the grid. In that case, the operation of observing the polarization direction of the needle projects the state vector $|\theta>$ onto the state vector |1> if the needle crosses the wire grid, onto the state vector |0> if the needle is stopped by the wire grid. A transition occurs from the intermediate state $|\theta>$ to one of the two definite valued states, when one performs this specific observation.

The presented analogy has imperfections. Firstly, the wires of the wire grid hardly represent the complexity of a molecular wire grid polarizer. Secondly, wire grid polarizers, like polyvinyl alcohol sheets, absorb electromagnetic radiation polarized parallelly to the wires, whereas in the analogy needles polarized perpendicularly to the wires are stopped. Thirdly, the analogy needs a complicated tensor pilot wave in order to implement the constraint $\theta=\varphi$ at the grid's plane. The steering value of the wave field at one point depends on the direction of the needle at that point. Through a more sophisticated treatment where the pilot wave also steers the phase of the basic constituents of the wire, there are possibilities to have it depend only of the coordinates at that point. This would however go beyond the scope of this paper, which intention is only to show that observations on macroscopic entities may be treated quantum mechanically.

3. Correlation measurements and the Bell test

In the light of the experimental results of Bell's inequalities tests, one could question the relevance of analogies akin the one presented in the preceding section. Indeed, the quantum mechanical correlation predictions for entangled systems conflict with predictions obtained via local classical deterministic models. Bell tests rule out such deterministic hidden variable theories, although some loopholes remain.

There are two points that should be considered concerning the presented analogy. Firstly, hidden variable models, as characterized by Clauser and Shimony, are aimed "to reinterpret quantum mechanics in terms of a statistical account of an underlying hidden-variables theory in order to bring it within the general framework of classical physics" [5]. As shown in the introduction, this analogy adopts a non-classical framework where measurement on systems in orthogonal states may yield the same result. The measurement of the *x*-coordinate may indeed be the same for two systems $|x_1>$ and $|x_2>$, provided that the interaction location with the detector is at the same point. In the example of section 2, the orthogonal states |1> and |0> could give the same measurement result, when the wire grid is set at an angle θ.

Secondly, Bell's argumentation takes advantage of a particular classical feature, the perfect correlation at angle 0 or π which for hidden variable theories results in a kink of the correlation function at those angles [6]. This kink is specific to classical local hidden variable theories, and inexistent in the considered analogy. To illustrate this, we consider two needles that got tangled up in their spinning motion and that separate in opposite directions with perpendicular rotational planes. At opposite sides, we execute the wire grid measurement with wire grids set relatively to each other at angle θ. We denote α the angle between the wire grid W_1 and the rotational plane of needle N_1 at one side; $(\alpha+\pi/2)$ is then the angle between W_1 and N_2's rotational plane. We then have the following probabilities:

$$\text{Prob}(N_1 \text{ passes } W_1) = \cos^2\alpha \tag{4}$$

$$\text{Prob}(N_2 \text{ passes } W_2) = \sin^2(\alpha+\theta) \tag{5}$$

The joint probability is then given by:

$$\text{Prob}(N_1 \text{ passes } W_1, N_2 \text{ passes } W_2) = \text{P}(1,1) = \cos^2\alpha \ \sin^2(\alpha+\theta) \tag{6}$$

If the wire grids are perpendicular and α is zero, there is perfect correlation equal to the quantum correlation and conflicting with Bell's inequality. However, the needles could separate with any angle α between 0 and π relatively to the analyzing orientation. Sampling over all orientations gives a joint probability of detection:

$$\Pr ob(1,1) = \frac{1}{4\pi}\int_0^{\pi} \cos^2\alpha \sin^2(\alpha+\theta)d\alpha = \frac{1}{8} + \frac{1}{4}\sin^2\theta \tag{7}$$

This prediction fits to the raw data of Aspect's 1982 test [7]. Similarly to the quantum mechanical predictions, (6) and (7) are stationary at angles 0 and π, conflicting with Bell's assumptions for local hidden variable theories [6].

4. Time evolution of arrows

In section 2, it was mentioned that $|\Psi(t+\Delta t)\rangle = \exp(i\omega\Delta t)\,|\Psi(t)\rangle$. With $\mathbf{\Psi}$ corresponding to the vector representing an arrow in real space, ω the angular spinning velocity and $\boldsymbol{e}_s$ the unitary vector representing the spinning axis, this corresponds for plane rotational motions to the solution :

$$\mathbf{\Psi}(t) = \mathbf{\Psi}(t{=}0)\cos\omega t + (\boldsymbol{e}_s \times \mathbf{\Psi}(t{=}0))\sin\omega t. \qquad (8)$$

However, freely rotating arrows have two rotational degrees of freedom. The rotational motion of an arrow must therefore be seen as the combination of two rotations:

- a *spinning* rotation $\omega_s\boldsymbol{e}_s$ of $\mathbf{\Psi}$ about a symmetry s-axis, represented by the unitary basis vector $\boldsymbol{e}_s$ perpendicular to $\mathbf{\Psi}$, also called the figure axis,
- a *precession* rotation $\omega_z\boldsymbol{e}_z$ of this figure axis about a fixed precession axis, which we take as the z-axis with basis vector $\boldsymbol{e}_z$.

Without external forces, ω_s and ω_z are constant and the s-axis delimitates a circular cone, with constant opening angle 2θ.

The time dependence $\mathbf{\Psi}(t)$ is given by operating successively two rotations $\mathbf{R}_s(\omega_s)$ and $\mathbf{R}_z(\omega_z)$ on $\mathbf{\Psi}(t{=}0)=\mathbf{\Psi}_0$. Using quaternion operators and given the initial parameters $\boldsymbol{e}_{s0}$, $\mathbf{\Psi}_0$ and $\boldsymbol{e}_z$, the kinematical specification of $\mathbf{\Psi}(t)$ yields:

$$\mathbf{\Psi}(t)=(\cos\tfrac{1}{2}\omega_z t,\ \boldsymbol{e}_z\sin\tfrac{1}{2}\omega_z t)(\cos\tfrac{1}{2}\omega_s t,\ \boldsymbol{e}_{s0}\sin\tfrac{1}{2}\omega_s t)\ \mathbf{\Psi}_0\ (\cos\tfrac{1}{2}\omega_s t,\ -\boldsymbol{e}_{s0}\sin\tfrac{1}{2}\omega_s t)(\cos\tfrac{1}{2}\omega_z t,\ -\boldsymbol{e}_z\sin\tfrac{1}{2}\omega_z t)$$

which reduces to:

$$\mathbf{\Psi}(t)= \cos\omega_z t\ \mathbf{\Psi}_s(\mathrm{t}) + \sin\omega_z t\ (\boldsymbol{e}_z \times \mathbf{\Psi}_s(\mathrm{t})) + 2\sin^2\tfrac{1}{2}\omega_z t\ (\boldsymbol{e}_z\,.\mathbf{\Psi}_s(\mathrm{t}))\ \boldsymbol{e}_z, \qquad (9)$$

with $\mathbf{\Psi}_s(\mathrm{t}) = \mathbf{\Psi}_0\cos\omega_s t + (\boldsymbol{e}_{s0} \times \mathbf{\Psi}_0)\sin\omega_s t$.

We verify that, for θ=0 or ω_z=0, the arrow is spinning in the plane normal to $\boldsymbol{e}_{s0}$ and equation (9) reduces to (8). In the more general case, for given θ and precession axis, there are four possible spinning modes, depending on the sign of ω_s and of ω_z: spinning up or down and positive or negative rotation of the spinning axis about the fixed precession axis. Furthermore, the projection of the spinning arrow on the z-axis has a constant value $\boldsymbol{e}_z.\boldsymbol{e}_{s0}$=cos$\theta$.

Coupling of both rotations with a pilot wave necessitates that they remain in phase with the pilot wave, through a condition: $qkx/t = m\omega_s = n\omega_z$, with q, m and n integer values and k the wavenumber of the pilot wave. Figure 2 illustrates two of the four modes where m=1 and n=2 and where the projection of $\omega_s\boldsymbol{e}_s$ on the z-axis is constantly half the value of ω_s. In order to obtain the two other modes, one inverts ω_s.

Needles spinning in these modes have noteworthy properties. When the needle has spun once about the spinning axis, the sign of $\boldsymbol{\Psi}$ is inverted: $\boldsymbol{\Psi}(t=2\pi/\omega_s)= -\boldsymbol{\Psi}_0$. It must spin twice in order to return to the original state. Two needles spinning in the same mode exclude each other at the same place since their identical motion causes them to collide when they come close. When however the two needles have opposite spins, while the precession rotations are oriented identically, their motions are compatible and they may spin one about the other when they are at the same place. Further investigation of these suggestive characteristics of spinning needles may shed light on the manner that particles aggregate or gain inertia.

5. Conclusion

Today there is some kind of consensus that there exist two domains where different rules apply. On the one hand, there is the classical domain based on familiar rules, on the other hand there is the quantum domain of molecular or submolecular constituents which obey rules that seem unsound with respect to classical principles. The presented analogy based on arrows, needles or rods, whose rotational motions are coherence with a pilot wave, suggest that the physics of ordinary objects may also be treated using the quantum-mechanical theoretical framework: complex vector state space with observation or evolution operators, superposition of states, Born's probability rule. Experiments on arrows, whether real or thought experiments, are conceptually easier to handle than experiments on fundamental particles and help to gain insight in the interpretation of quantum physics. After all, "all we do is draw little arrows on a piece of paper – that's all!"[8].

References

[1] J. S. Bell, *Against Measurement*, *Phys. World*, pp. 33-40 (Aug. 1990)

[2] W. Heisenberg, *The physical content of quantum kinematics and mechanics*, in *Quantum Theory and Measurement*, translated and re-edited by J. A. Wheeler and W. H. Zurek, W. H. (Princeton University Press, Princeton, New Jersey), 1983, p.64.

[3] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford University Press, Oxford), 1958, p.35.

[4] A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.*, **47** (1935) 777.

[5] J. F. Clauser and A. Shimony, *Rep. Prog. Phys.*, **41** (1978) 1881.

[6] J. S. Bell, *Einstein-Podolsky-Rosen Experiments*, in *Proceedings of the Symposium on Frontier Problems in High Energy Physics* (Pisa), 1976, p.33-45.

[7] A. Aspect, *Trois tests expérimentaux des inégalités de Bell par mesure de corrélation de polarisation de photons* (PhD thesis No. 2674, Université de Paris-Sud, Centre d'Orsay, 1983), as reported by C. Thompson, quant-ph/9704045.

[8] R. P. Feynman, *QED: The Strange Theory of Light and Matter* (Princeton University Press, Princeton, New Jersey), 1985, p.24.

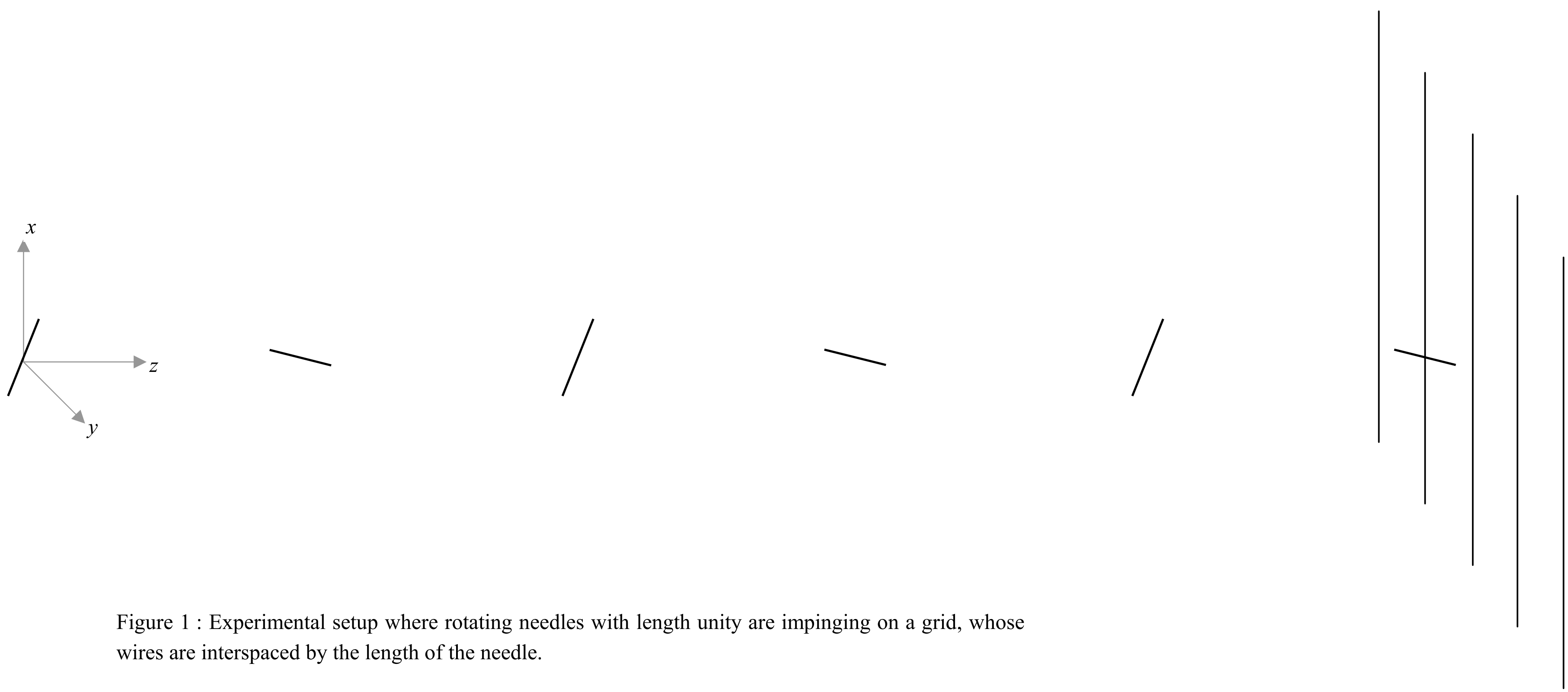

Figure 1 : Experimental setup where rotating needles with length unity are impinging on a grid, whose wires are interspaced by the length of the needle.

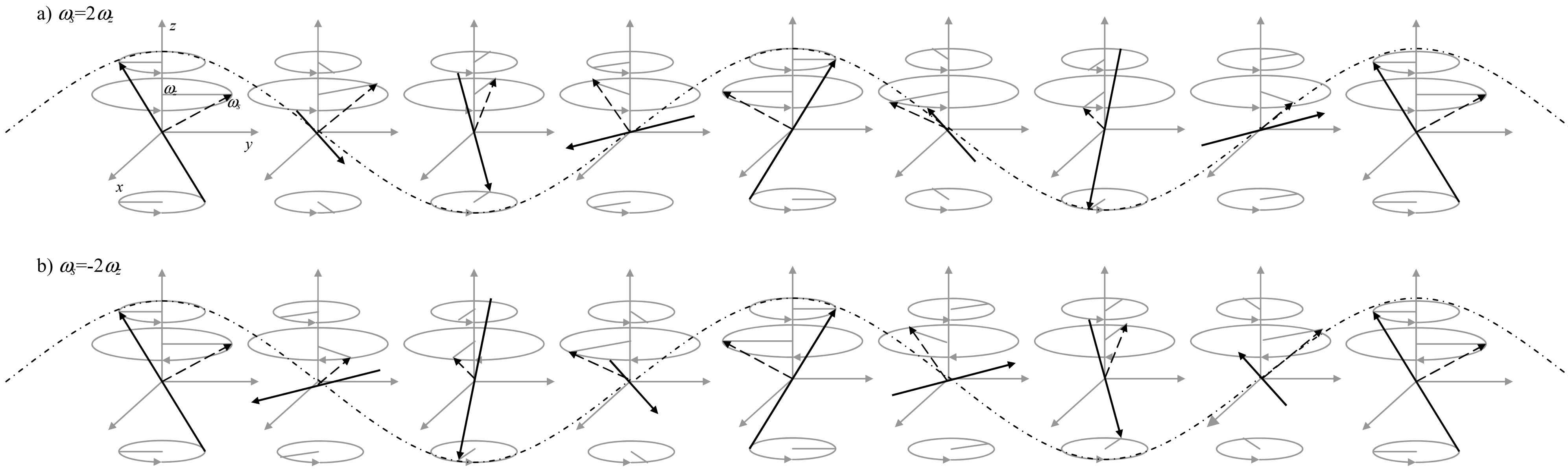

Figure 2 : Rotational motion of arrows, composed of a spinning motion ω_s and a precession motion: a) $\omega_z=\omega_s/2$ and b) $\omega_z=-\omega_s/2$.